SHRI ANGALAMMAN COLLEGE OF ENGINEERING AND TECHNOLOGY
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DEPARTMENT OF MATHEMATICS

MA1301 Discrete Mathematics

UNIT-I
PART-A

1.Give the converse and the cotrapositive of the implication " If it is raining then I get wet"

2. verify whether $(P \vee Q) \rightarrow P$ is a tautology.
3.Prove that (R $(R \rightarrow S) \vee \neg(R \rightarrow S)$ is a tautology.
4.State Demorgan Laws of logic.
5. Prove that $P \rightarrow Q$ and its contrapositive $\neg Q \rightarrow \neg P$ are equivalent
6. Obtain the PDNF of $\neg P \vee Q$.
7.When a set of formulae is inconsistent and consistent.
8. Define PCNF of a statement.
9.Determine whether the conclusion C follows logically from premises H_1 and H_2 or not

$$H_1 : P \rightarrow Q, H_2 : P, C : Q$$

PART-B

1. obtain the PCNF and PDNF of $(\neg P \rightarrow R) \wedge (Q \rightarrow P)$
2. Show that the following premises are inconsistent

$$E \rightarrow S, S \rightarrow H, A \rightarrow 74andE \wedge A$$

3. If there was a ball game , then traveling was difficult. If they arrived on time, then traveling was not difficult. They arrived on time . Therefore, there was no ball game , Show that these statements constitute a valid argument.

4.Using truth table show that $P \rightarrow (Q \vee R) and (P \rightarrow Q) \vee (P \rightarrow R)$ are logically equivalent.
5. Obtain the PCNF and PDNF of $(\neg P \rightarrow Q) \wedge (P \rightarrow R) \wedge (R \rightarrow P)$
6. Using indirect proof show that D can be derived from

$$(A \rightarrow B) \wedge (A \rightarrow C) \neg (B \wedge C) \,\&\, DVA$$

7.Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises

$$P \vee Q, Q \rightarrow R, P \rightarrow S \,\&\, \neg S.$$

8.. Prove that $(x)(P(x) \rightarrow Q(x)), (x)(R(x) \rightarrow \neg Q(x)) \Rightarrow (x)(R(x) \rightarrow \neg P(x))$
9.Show that $\neg(P \wedge \neg Q)$ **,** $\neg Q \vee R$ **,** $\neg R \Rightarrow \neg P$

10.Show that the premises $A \rightarrow (B \rightarrow C), D \rightarrow (B \wedge \neg C), A \wedge D$ are inconsistent**.**

11 If there was a ball game, then traveling was difficult. If they arrived on time , then traveling is not difficult. They arrived on time, therefore there was no ball game.

Test the validity of the above Argument.

12 Without constructing Truth table Verify whether $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is contradiction (or) Tautology.

13.Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$ bi conditional $\Leftrightarrow$ R

14.Show that $(R \wedge S)$ can be derived from the premises $P \to Q, Q \to \neg R, R, P \vee (R \wedge S)$.

15.Without constructing Truth table Obtain PDNF of $(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$ and Hence find it's PCNF

16. Construct the truth table for the formula $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$.

17.Show that $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology.

18.Show that RVS is a valid conclusion from the premises $C \vee D, (C \vee D) \to \neg H, \neg H \to (A \wedge \neg B), (A \wedge \neg B) \to (R \vee S)$.

19.Without using Truth table find the PCNF form of $(P \wedge Q) \vee (\neg P \wedge R)$.

20. Determine whether the compound proposition $\neg(q \to r) \wedge r \wedge (p \to q)$ is tautology (or) contradiction .

21.Show that d can be derived from the premises $(a \to b) \wedge (a \to c), \neg(b \wedge c), (d \vee a)$

22.Using rule of inference show that SVR is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$

23.Using cp or otherwise obtain the following implication $(x)(P(x) \to Q(x)), (x)(R(x) \to \neg Q(x)) \Rightarrow (x)(R(x) \to \neg P(x))$

24.Without constructing truth table , obtain PCNF of $(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \Leftrightarrow \neg R))$ and hence find its PDNF

25. Show that the expression $(P \vee Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology by using truth table.

26.Obtain the product of sums canonical form for $(P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$

27. Show that $J \wedge S$. Logically follows from the premises $P \to Q, Q \to \neg R, R, P \vee (J \wedge S)$

UNIT –II Predicate Calculus

Part-A

1. Let the universe of discourse be E={ 5,6,7}.Let A={ 5,6} and B={ 6,7}. Let P(x): x is in A; Q(x): x is in B and R(x,y):x+y<12. Find the truth value of

$$((\exists x)P(x) \to Q(x)) \to R(5,6)$$

2. Give an example in which $(\exists x)(P(x) \to Q(x))$ is true but $(\exists x)P(x) \to (\exists x)Q(x)$ is false.

3. Express the statement , "Some people who trust others are rewarded" in symbolic form

4. Give an example of free and bound variables in predicate logic

5. Write the statement ,"every one who likes fun will enjoy each of these plays"in symbolic form.

6. Define bound and free variables.

7. Symbolize the following statement with and without using the set of positive integers as the universe of discourse. "Give any positive integer, there is a greater positive integers."

8. Symbolize the expression ," For any x and any y, if x is taller than y, then y is not taller than x"

9. write the scope of the quantifiers in the formula $(x)(P(x) \to (\exists y)R(x,y))$

10 . Express the statement " for every x" there is exist a "y" such that $x^2 + y^2 \geq 100$ in symbolic form

11. Write the expression in English $\forall n(q(n) \to p(n))$. Where p(n): n is an evebn integer and q(n):n is divisible by2 within the universe of all integers.

PART-B

1.Show that $(x)(P(x) \to Q(x)) \wedge (x)(Q(x) \to R(x)) \Rightarrow (x)(P(x) \to R(x))$

2.Prove that (i) $(\exists x)A(x) \to (x)B(x) \Leftrightarrow (x)(A(x) \to B(x)$

(ii) $(\exists x)(A(x) \to B(x)) \Leftrightarrow (x)(A(x) \to (\exists x)B(x)$

3. Show that the premises," A student in this class has not read the book" and " Every one in this class passed the first examination" imply the conculusion " Some who passed the first examination has not read the book".

4. using CP rule Show that, $(x)(P(x) \rightarrow Q(x)) \Leftrightarrow (x)(P(x)) \rightarrow (x)(Q(x))$

5. Show that $(\exists x)M(x)$ follows logically from the premises

$$(x)(H(x) \rightarrow M(x)) and (\exists x)H(x)$$

6. Using indirect proof, $(x)P(x) \vee Q(x) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$

7. Prove that $(x)(P(x) \rightarrow Q(x)), (x)(R(x) \rightarrow \neg Q(x)) \Rightarrow (x)(R(x) \rightarrow \neg P(x))$

8. Prove that $(\exists x)A(x) \wedge B(x) \Rightarrow (\exists x)A(x) \wedge (\exists x)B(x)$

9. Show that $(\forall x)(P(x) \rightarrow Q(x)) \wedge (\forall x)(Q(x) \rightarrow R(x)) \Rightarrow (\forall x)(P(x) \rightarrow R(x))$

10.Show that $(\exists x)P(x) \rightarrow\Rightarrow (\forall x)Q(x) \Leftrightarrow (\forall x)(P(x) \rightarrow Q(x))$

11.Use indirect proof, show that $(\exists Z)Q(Z)$ is not valid conclusion from the premises

$(\forall x)(P(x) \rightarrow Q(x)$ and $(\exists y)Q(y)$

12.Using CP or otherwise obtain the following implication

$(x)(P(x) \rightarrow Q(x)), (x)(R(x) \rightarrow \neg Q(x)) \Rightarrow (x)(R(x) \rightarrow \neg P(x))$

13.Verify the validity of the following arguments- Lions are dangerous animals. There are lions. Therefore, there are dangerous animals**)**

14. Prove that the implication $(\forall x)(P(x) \rightarrow Q(x)), (\forall x)(R(x) \rightarrow \neg Q(x)) \Rightarrow (\forall x)(R(x) \rightarrow \neg P(x))$

15.Express the negations of the following statement using quantifiers and in statement form

" No one has done every problem in the exercise"

16. Prove that $(\forall x)((P(x) \rightarrow Q(y) \wedge R(x))), (\exists x)(P(x) \Rightarrow Q(y) \wedge (\exists x)(P(x) \wedge R(x))$

17.Show that $(x)((P(x) \rightarrow Q(x)) \wedge (x)(Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \rightarrow R(x))$

18. Show that $\neg P(a,b)$ follows logically from $(x)(y)(P(x,y) \rightarrow W(x,y))$ and $\neg W(a,b)$

19. Prove that $(\exists x)((P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)))$

20.Prove that $(\exists x)A(x) \rightarrow B \Leftrightarrow (x)(A(x) \rightarrow B)$

21.Prove that $(x)(H(x) \rightarrow A(x) \Rightarrow ((\exists y)(H(y) \wedge N(x,y)) \rightarrow (\exists y)(A(y) \wedge N(x,y))$

22. Show that $(x)(P(x) \vee Q(x)) \Leftrightarrow (x)(P(x) \vee (\ni x)Q(x))$.

23. Write thwe symbolic form of

(i) x is the father of the mother of y'

(ii) " Given any positive integer, there is a positive integer" with and without using set of positive integers as the universe of disclosure

UNIT-III SET THEORY

PART- A

1. If R ={(x,y)/ x>y} is a relation on X = {1,2,3,4}, draw the graph of Rs
2. Prove that in a Boolean algebra; x +x**.**y =x

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3. Draw the hasse diagram of the poset $(P(A), \leq)$ where A = {x,y,z}
4. Suppose that the sets A and B have m and n elements respectively. How many elements of $A \times B$? How many different relations are there from A to B?
5. Let D(45) denote the set of all positive divisors of 12. Draw the Hasse diagram of D(45)

6 .If A ={1,2,3,4} and R ={ (1,1),(1,3),(2,3),(3,2),(3,3),(4,3)}, determine the matrix of the relation R.

7. If A ={{1,2},{3}},B={{1},{2,3}} and C ={{1,2,3}}, then show that A,B,C are mutually disjoint.

8. Give an example of two-element Boolean algebra

9. In the following lattice find $(b_1 \oplus b_3) * b_2$

10. If R = {(1,1),(1,2),(2,3) } and S ={(2,1),(2,2),(3,2)} are relations on the set A = {1,2,3}, Verify whether $R \circ S = S \circ R$ by finding the relation matrices of $R \circ S = S \circ R$

11. Partition A ={0,1,2,3,4,5} with minsets generated by B_1 = {0,2,4} and B_2 = {1,5}.

12.If a poset has a least element, then prove it is unique.

13. The following is ther Hasse diagram of a partially ordered set. Verify whether it is a lattice.

14**.** Given A = { 1,2,3,4} AND R = { (1,2),)1,1), (1,3), (2,4), (3, 2)} and S = { (1,4,(1,3), (2,3), (3,1), (4,1)} are relations in A. Find S·R

15.If a and b are two elements of Boolean algebra , Prove that a + (a,b) = a

PART-B

1. How many positive integers not exceeding 500 are divisible by 2,3,or 5 ?
2. Find the equivalence relation on the set S = { 1,2,3,4,5 } which generates the partition {1,2,3,4,5}. Also

Draw the graph of the relation.

3.Define lattice. Prove that in a lattice for any $a,b,c \in L, a \oplus (b * c) \le (a \oplus b) * (a \oplus c)$

4. If any Boolean algebra, show that (a+b)(a'+c) = (ac +a'b) =ac+a'b+bc.

5.Prove that, for any three sets A,Band C

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

6. Prove that R = {(1,1),(1,4),(4,1),(4,4),(2,2),(2,3),(3,2),(3,3)} is an equivalence relation. Also write the matrix of **R** and Sketch its graph.

7 . Show that, in a lattice if $a \le b and c \le d, then a * c \le b * d . and a \oplus c \le b \oplus d$.

8. Prove Demorgan laws in a Boolean algebra.

9. Let N be the set of all natural numbers with the relation R as follows : a R b if and onlyif a divides b, Show that R is a Partial order relation on N

10.Show that every distributive lattice is modular.Is the converse true ? Justify the claim.

11. Let R be the relation on A ={1,2,3} such that (a,b) if and only if a+b is even, Find the relational matrix of R,R^{-1}, R and R^2

12.Show that in a complemented distributive lattice, the Demorgan's laws hold.

13.If R ={(1,1),(1,2),(1,3),(2,4),(3,2)}and S ={(1,3),(1,4),(2,3),(3,1),(4,1)} are the relations on A = {1,2,3,4}, find the relation $S \circ R$ by using relational matrix.

14. In a Boolean algebra B, prove that $(a \wedge b)' = a' \vee b' and (a \vee b)' = a' \wedge b' for all a,b \in B$

15. If R is the relation on the set of positive integers such that $(a,b) \in R$ iff a^2 +b is even then Prove that R is an equivalence relation.

16. In a lattice $(L, \le), prove that X \vee (Y \wedge Z) \le (X \vee Y) \wedge (X \vee Z)$

17. Let X = {1,2,3,4……7} and r{(x-y)/ x-y is divisible by 3}. Show that R is a equivalence relation and draw the graph of R.

18. Let (L , ≤) be a lattice in which * and ⊕ denote the operations of meet and join respectively. For any $a,b \in L.$ Prove that $a \le b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$

19. Prove that every chain is distributive lattice.

20. prove that lub anf glb of any two elements are unique on a poset.

UNIT –IV FUNCTIONS

PART –A

01. List all possible functions from X = {a,b,c} to Y = {0,1} and indicate in each case whether the whether the function is 1-1, onto.
02. If A={1,2,…n} show that any function from A to A, which is one to one must also be onto and conversely.
03. If A has m elements and B has n elements, how many functions are there from A to B?
04. Let $f : R \rightarrow R$ and $g : R \rightarrow R$ where R is the set of real numbers find f.g and g.f, if f(x) = x^2-2 and g(x) = x + 4
05. Show that the function $f(x) = x^3$ & $g(x) = x^{1/3}$ for $x \in R$ are inverse of one another.
06. Let f,g,h are functions from N to N where N is the set of natural numbers so that f(n) = n+1,g(n) = 2n, h(n) = { 0 n is even
{1 n is odd

Determine the f.f, f.g, g.f and (f.g).h
07. The inverse of the inverse of a function is the function itself.
08. Find all the mappings from A = {1,2} to B={3,4}
09. Let $h(x,y) = g(f_1(x,y), f_2(x,y))$ for al positive integers x and y ,where $f_1(x,y) = x^2 + y^2$, $f_2(x,y) = x$ and $g(x,y) = xy^2$ find h(x,y) in terms of x and y
10. If χ_A denotes the characteristic function of the set A, Prove that $\chi_{A-B}(x) = \chi_A(x) - \chi_{A\cap B}(x)$ for all $x \in E$ the universal set.
11. If A has 3 elements and B has 2 elements, how many functions are there from A to B.
12. Define Characteristic function.
13. If $f:A \rightarrow B$ and $g:B \rightarrow C$ are mappings and $g \circ f : A \rightarrow C$ is one to one Prove that f is one to one.
14. If χ_A denotes the characteristic function of the set A, Prove that $\chi_{A\cup B}(x) = \chi_A(x) + \chi_B(x) - \chi_{A\cap B}(x)$ for all $x \in E$, the universal set.
15. Determine whether the permutation $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$ is even or odd.
16. Prove that if A and B are ant two subset of a universal set U then $X_{A\cap B}(x) = X_A(x).X_B(x)$.

PART-B

01) If f and g are bijections on a set A, Prove that $f \circ g$ is also a bijection

02) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 4 & 2 & 1 \end{pmatrix}$ are permutations on the set A={1,2,3,4,5}, find a permutation g on A such that $f \circ g = h \circ f$

03) The Ackerman function A(x,y) is defined by is defined by
A(x,y) = y+1, A(x+1,0) = A(x,1), A(x+1,y+1) = A(x,A(x+1,y)) Find A(2,1)

04) Let a<b. If f:[a,b] $f : [a,b] \to [0,1]$ is defined by $f(x) = \frac{x-a}{a-b}$ Prove that f is a bijection and find it inverse (Here [a,b] and [0,1] are closed intervals)

05) If R denotes the set of real numbers and $f : R \to R$ is given by $f(x) = x^3 - 2$, find f^{-1}

06) Show that f :R-[3] $\to$ R-[1] given by $f(x) = \frac{x-2}{x-3}$ is a bijection.

07) Let f (x) = x+2, g(x) = x-2 and h(x) = 3x for $x \in R$. Find $g \circ f$ and $f \circ h \circ g$

08) Let D(x) denote the number of divisors of x. Show that D(x) is a primitive recursive function.

09) Prove that the set 2P of even positive integers has the same cardinality as the set P of positive integers.

10) If $f:A \to B$, $g:B \to C$ and $h:C \to D$ are functions then prove $h \circ (g \circ f) = (h \circ g) \circ f$

11) Define even and odd permutations. Show that the permutations

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 5 & 7 & 8 & 6 & 1 & 4 & 3 \end{pmatrix} \text{ and } g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$$

are respectively even and odd.

12) Let the function f and g be defined f(x) = 2x + 1 and $g(x) = x^2 - 2$. Determine the composition function $f \circ g$ and $g \circ f$

13) Let a and b be positive integers and suppose Q is defined recursively as follows

$$Q(a,b) = \begin{cases} 0 & \text{if } a < b \\ Q(a-b,b)+1 & \text{if } b \le a \end{cases}$$

Find Q(2,5), Q(12,5) Q(5861,7)

14) Let $f : R \to R$ be defined by f(x) = 2x-3 Find a formula for f^{-1}

15) Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by using Characteristic function.

16)Find all mappings from A = 1,2,3} to B = {4,5} find which of them are one to one and which are on to

17) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ are permutations, prove that

$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

UNIT-V GROUPS

PART-A

1. If P(S) is the power set of a non-empty set S, prove that (P(S),n) is a monoid.

2. Give an example of semi group which is not a monoid.

3.Find the minimum distance between the code words x = (1,0,0,1), y =(0,1,0,0) and z = (1,0,0,0)

4.Show that the inverse of an element in a group (G,*) is unique

5. Find the minimum distance between the code words x = (1,0,0,1), y =(0,1,1 ,0) and z = (1,0,1 ,0)

6. Define Group Code.

7. Define sub semi-group with an example.

8. If the minimum distance between two code words is 5, how many errors can be detected and how many errors can be corrected?

9.Find a sub- group of order two of the group (Z_8 ,$+_8$)

10.State Lagrange's theorem for finite groups.

11..Show that semi- group homomorphism preserves the property of idempotency.

12. Define normal subgroup.

13. Let x = 1001, y= 0100, z = 1000 .Find the minimum distance between the code

PART-B

1.State and prove Lagrange's theorem.

2. Define Normal sub group . Prove that the Kernel of a homomorphism f from a group (G, *) to (H, Δ)

3. Find all the subgroups of (Z_{12} , $+_{12}$)

4. Obtain the single – error correcting code generated by the parity-check

matrix $H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

5.Define submonoid with an example. For any commutative monoid (M, *), prove that the set of idempotent elements of M forms a submonoid.

6. Show that H = {[0],[4],[8] } is a sub group of (Z_{12} , $+_{12}$). Also find the left cosets of H in (Z_{12} , $+_{12}$).

7. Prove that the intersection of two normal subgroups of a group G is also a normal subgroup.

8.Show that the (2,5) encoding function $e:B^2 \rightarrow B^5$ defined by e(00) = 00000,e(01) =01110, e(10) = 10101 and e(11) 11011 is a groups code. Find also the minimum distance of this group code.

9.Show that the order of a subgroup of a finite group G divides the order of the group G.

10.Find the code words generated by the parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$ **,** when the encoding function is $e:B^3 \rightarrow B^6$

11. Let H be a nonempty subset of a group (G , *). Show that H is a subgroup of G if and only if $a * b^{-1} \varepsilon H$ for all a,b ε H

12.Show that the Kernel of a group homomorphism is a normal subgroup of a group.

13. If S = $N \times \mathbf{N}$ **,** the set of ordered pairs of positive integers with the operation * defined by (a,b) * (c,d) = (ad +bc,bd) and if $f:(S,*) \rightarrow (Q,+)$ is defined by f(a,b) =a/b, show that f is a semigroup homomorphism**.**

14. Show that every sub group of a cyclic group is cyclic.

15. Let $$H=\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ be a parity check matrix. Determine the group code $l_H : B_2 \rightarrow B^5$